TIME-FREQUENCY SIGNATURES THROUGH OPTICAL DIFFRACTION SCANNING

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ACOUSTICS AND SEISMICS LABORATORY
Institute of Science and Technology
THE UNIVERSITY OF MICHIGAN

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NOTICES

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ABSTRACT

An optical scanning method has been developed to present frequency content as it occurs and changes throughout a variable-density seismogram. Energy distributions are shown in both frequency and time. A two-dimensional pattern is presented which can be visually evaluated, and is suitable for numerical measurement. This report describes and discusses the method, and gives illustrative examples. It also describes a similar scanning method by which seismic wave dispersions can be revealed and measured.

1
INTRODUCTION

Diffraction is caused by an obstacle placed in the path of a plane wave of monochromatic light [1, 2, 3]. The diffraction pattern formed at an infinite distance has the distribution of the Fourier transform of the geometry of the obstacle. In practice, the distribution of the Fourier transform, called Fraunhofer diffraction, is found along the focal plane of a condensing lens placed in the path of the diffracted light. The basic equation contains the Fresnel-Kirchhoff integral, which has the same form as the Fourier transform.

This phenomenon is used by placing a variable-density seismogram in the path of a plane wave of nearly monochromatic light. If the opacity of the variable-density record is a particular function of the voltage output of the seismometer, the energy distribution of the resulting diffraction defines the energy-density spectrum of the seismogram. Since a variable-density seismogram will vary in density along a single dimension, the seismic information will be within a narrow channel.

Basically, diffraction is used by placing the entire length of the seismogram within an aperture, and photographing or scanning the resulting pattern; by this means the entire frequency distribution of the seismogram is found, but the times of occurrence of the various frequency concentrations are lost; that is, phase is lost. Thus, for instance, frequency concentrations

due to P waves cannot be distinguished from those due to surface waves, other than by prior knowledge of the frequency bands of each.

To retain time of occurrence of energy at various frequencies a scanning method was devised as follows. The seismogram was moved horizontally past an aperture in the light path while a viewing camera was moved vertically. The viewing camera was focused on a narrow horizontal slit in the diffraction plane. In the resulting photograph the frequency axis is horizontal and the time axis is vertical; energy is represented by film density, and can be located in time and frequency. Qualitatively, a pattern is presented for judgment through human eyes. Quantitatively, a pattern can be scanned in any direction by use of a microphotometer.

Another method of scanning might be termed dispersion scanning. Dispersed seismic waves show up in variable-density seismograms as a continuous change in spatial frequency. Since spatial frequency variations of this kind cause light energy to focus, placing such an obstacle in a lens system changes the diffractive focal distance. Diffraction energy concentrations occur in front of the condensing lens focal plane on one side of center, and behind the focal plane on the other side of center. Dispersion can then be discovered and measured by focusing the viewing camera before and behind the focal plane. In this type of scanning the viewing camera is elevated while the image is slowly brought through the focal plane of the viewing camera. In the resulting photograph the horizontal axis is again frequency, the vertical axis is focal position, and the positions of the energy concentrations indicate the amount of dispersion.

2
EXPERIMENTAL SETUP FOR TIME-FREQUENCY SIGNATURES

Figure 1 is a sketch of the components needed for time-frequency scanning. A plane wave of nearly monochromatic light is obtained by placing a high-pressure mercury vapor lamp at the focus of a collimating lens. An interference filter centered at 5461 Å is placed in the resulting collimated light path. A condensing lens is positioned so that the image of the luminous mercury falls upon a 5-μ slit. A second lens, whose focal plane coincides with the slit, acts as a collimator. The required plane wave is thus produced.

A small aperture of any desired shape is placed in the light path directly in front of a variable-density film which can be transported horizontally across the aperture. An objective lens focuses the light emerging through the aperture and the variable-density film. Fraunhofer diffraction is produced.

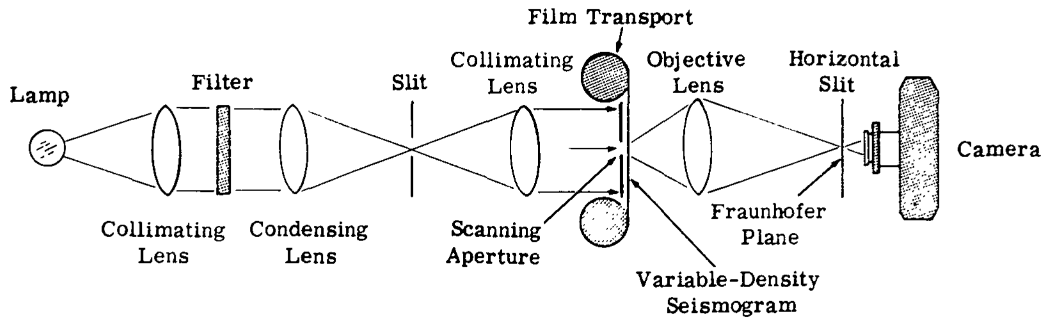


FIGURE 1. TOP VIEW OF TIME-FREQUENCY SIGNATURE SETUP

An adjustable horizontal slit is located at the Fraunhofer diffraction plane, so that vertically diffracted and scattered light is removed from the image, and so that the diffraction pattern can be contained in a narrow channel. A microscope with attached camera is used to view this slit. The microscope is on a motor-driven platform which can be moved vertically.

When the variable-density seismogram is moved horizontally in front of an aperture, the portion of the seismogram causing the diffraction pattern is constantly changing. Thus at any given instant the diffraction pattern in the plane of the narrow horizontal slit is that of the portion of the seismogram in front of the aperture.

As the seismogram is drawn horizontally across the aperture, the microscope is raised. These two movements combine so that a particular vertical position of the camera film corresponds to a particular portion of the seismogram. A continuously changing diffraction pattern is recorded, the resolution of which is limited by both the width of the aperture and the width of the horizontal viewing slit. Frequency components are recorded as they develop and change throughout the seismogram.

Figure 2 is a photograph of the equipment used.

3
THE SIGNATURE PATTERN

A one-dimensional diffraction pattern consists of a central bright image of the slit, and side bands symmetrically located on either side of center. The distance of the side bands from the center is related to frequency in a closely linear manner. For instance, if a sinusoidal

spatial frequency of 5 cycles/mm diffracts so that a line occurs 1 mm on each side of center, a sinusoid of 10 cycles/mm would diffract so that a line forms at a distance of 2 mm on each side of center. At the other extreme, if one diffracted a Δ-function, which contains all frequencies, a uniform density would be found on either side of the central image. Indeed, a narrow slit or a pinhole (both approximations of a Δ-function) is employed optically to distribute the light evenly.

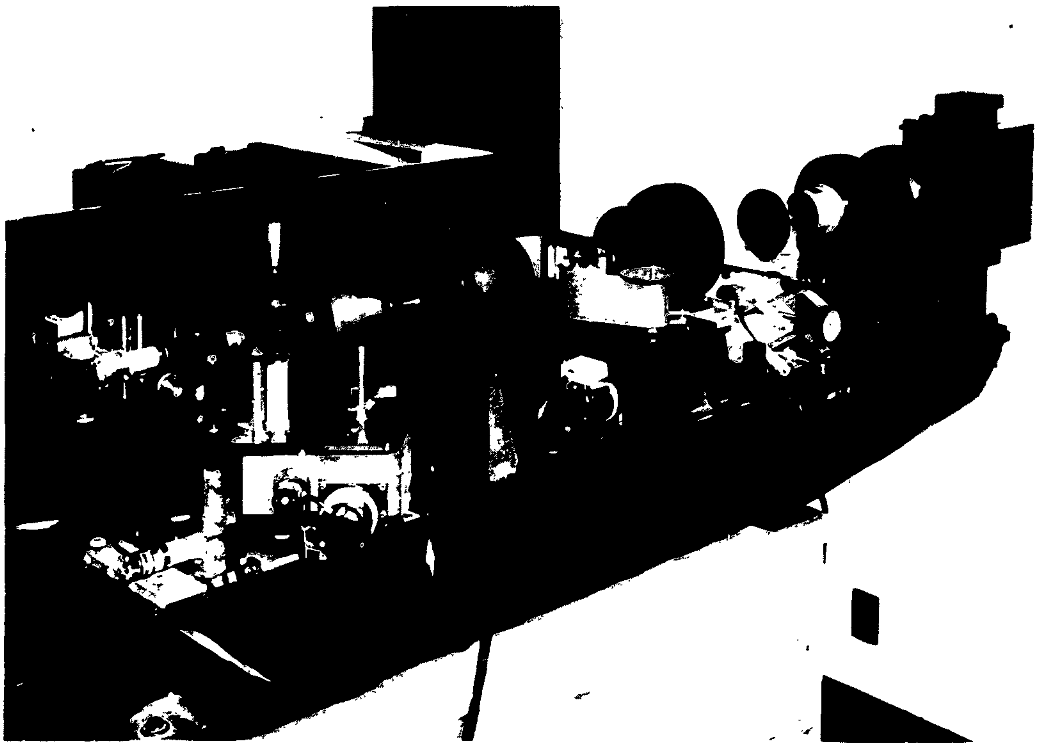
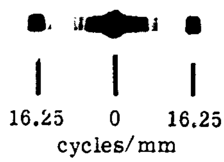


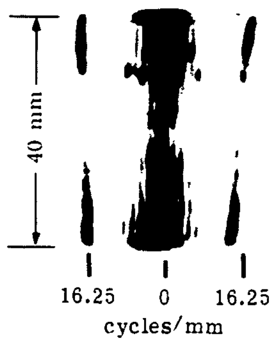
FIGURE 2. SETUP FOR OBTAINING TIME-FREQUENCY SIGNATURES AND DISPERSION SCANS

Figure 3(a) shows a variable-density record of a linearly changing sinusoid of 248 cycles, varying from $\lambda_1 = 0.171$ mm at one end of the record to $\lambda_2 = 0.148$ mm at the other. In Figure 3(b) two bands of approximate width $\lambda_1 - \lambda_2$ are seen on either side of the bright central image. Note that no higher diffraction orders can be distinguished in this pattern. For a sinusoid a

(a)



(b)



(c)

FIGURE 3. DIFFRACTIVE OPERATIONS ON AN ARTIFICIAL RECORD. (a) Variable-density record of a sinusoid whose frequency is changing slowly. (b) Diffraction pattern. (c) Time-frequency signature obtained from this record.

single order only would be expected. The record was scanned with a rectangular aperture with a width of 10 average wavelengths of the record. Aperture scan was from the low-frequency end of the record to the high-frequency end. The viewing camera scan proceeded from the bottom of the photograph to the top. Thus at the bottom of Figure 3(c) the loci of the diffracted light are closer to the central image than at the top. Straight line loci indicate a linear change in frequency. As the locus of the diffraction as found in the time-frequency signature is frequency as a direct function of time, the first derivatives of these curves define dispersion.

In masking the record shown in Figure 3(a) the center portion of the mask was inadvertently made narrower than the two ends, with the result that the data channel is narrower near the middle of the record. As light for diffractive purposes is gathered across the width of the channel, narrowing the channel reduces the energy available for diffraction. Such narrowing would show up in the diffraction pattern as a reduction in energy of the signal. Energy reduction is seen in Figure 3(c) by the decrease in density near the time center of the record.

The poorly masked record was used for this memorandum because it illustrates how a record can be weighted. In this case the weighting was accomplished by narrowing the channel, which is exactly the same as narrowing the aperture. An aperture can be made in any desired shape, so that any desired weighting can be made.

Parallel lines close to the central image are caused by aperture diffraction. Film thickness variations acting as wedges cause the lines to be irregularly wavy.

4
SCANNING APERTURE

A variable-density seismogram — a spatial record of a time phenomenon — can be analyzed as if its single direction were time instead of space. A viewing aperture is spatial, but extends across the seismogram; so it, too, can be viewed as temporal. Hence, a discussion in terms of "time window" scanning and time frequency is valid for this spatial device.

A fundamental limitation of scanning arises from the relationship between the size of the aperture and the resolution of the frequency. When a long record is scanned with a small aperture, the frequency indication at any instant is a result of analyzing the boundaries of the aperture in combination with the portion of the record visible through the aperture. The narrower the aperture, the more it obtrudes in the frequency analysis. Thus in scanning through time for changes in frequency we find ourselves at cross purposes: fine frequency resolution requires

a large aperture, meaning that the frequency is averaged over a comparatively long time; but fine time resolution requires a narrow aperture, which destroys fine frequency resolution. As pointed out before, the narrowest aperture would approximate a Δ-function. With such an aperture all scanned records would show all frequencies; in fact, the original time-function could be recovered by any vertical trace on a time-frequency signature, and hence no new information could be found. A variable-density seismogram would act only as a light attenuator.

Since frequency is defined as the variation in time of a function, elapsed time is necessary for its interpretation. One approach is as follows:

$$f = n/\Delta t \tag{1}$$

where n = number of cycles
f = frequency
Δt = elapsed time

If the number of cycles is restricted to a whole number, then the possible error, or "spread," is $\Delta f = 1/\Delta t$, where Δf is the defined bandwidth, because the counting can be in error by $\pm 1/2$. Similarly, if a Fourier transform of a function is taken over a time Δt, we find that

$$\Delta f \, \Delta t = k \tag{2}$$

where k is a constant
Δf is the width of the frequency band between arbitrarily chosen cut-off points

Of course, for the best frequency resolution k should be made as small as possible.

In a manner roughly analogous to counting by fractions of a cycle in Equation 1, the constant k in Equation 2 can be controlled to some extent by weighting the aperture or time window. (This process is sometimes called tapering in digital computing, and is known as apodizing in optics.) In short, a Gaussian aperture is optimum for time-frequency resolution. A lucid discussion of time-frequency resolution is found in Reference 4, pp. 62-73.

In our optical system weighting corresponds to the shape of the aperture. To illustrate the diffraction caused by an aperture five different apertures were prepared, and their diffraction patterns recorded. Figure 4 shows the patterns obtained from two rectangular apertures, two approximately Gaussian apertures, and a triangular aperture. These diffraction patterns reveal energy density spectra as approached through the Fourier transform. The two rectangular apertures correspond to a square wave pulse, and the resulting diffraction reveals the orders or Fourier components. The two Gaussian apertures cause Gaussian diffraction, as this function transforms into itself.

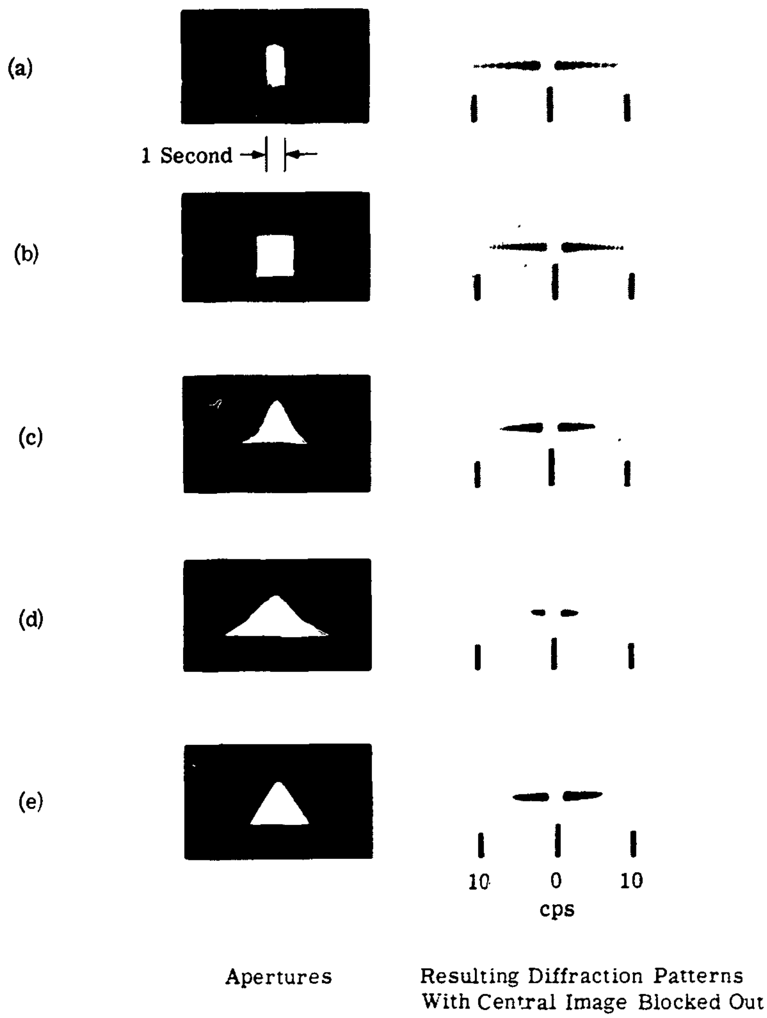


FIGURE 4. DIFFRACTION CAUSED BY VARIOUS APERTURES

The effects of the size and shape of the scanning aperture upon time-frequency signatures have been dwelt upon for two reasons. First, they are often ignored in presenting time-varying frequency analysis, even though they are of great importance. Second, they are obvious when this optical scanning method is used, and require explanation.

5
TIME-FREQUENCY SIGNATURES OF A SEISMOGRAM

A seismogram in graphical form with several bandpasses obtained with electronic bandpass filtering is shown in Figure 5. The seismogram in variable-density form, and five time-frequency signatures (one for each of the five apertures shown in Figure 4), are shown in Figure 6.

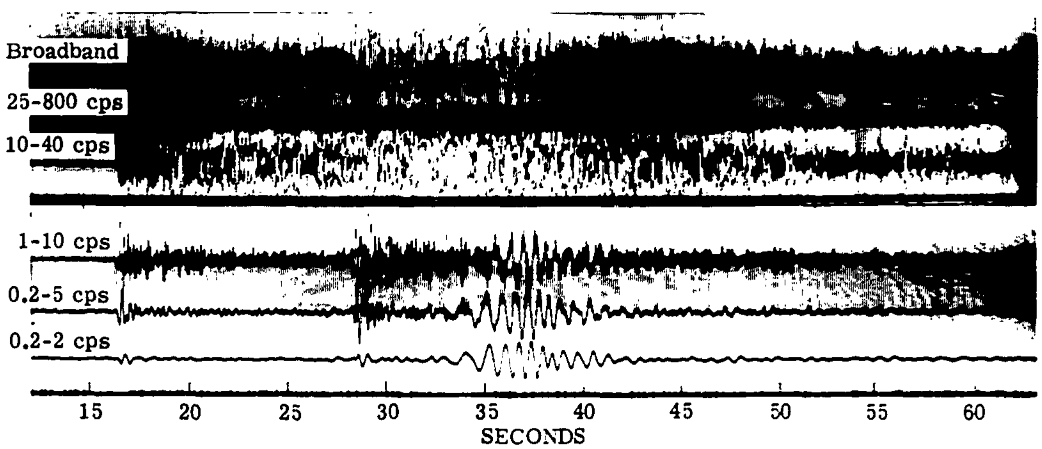


FIGURE 5. SEISMOGRAM AND VARIOUS BANDPASS FILTERINGS

This seismogram was chosen primarily because it had already been frequency analyzed by other methods (bandpass filtering, analog computation, and digital computation). The results from optical analysis could then be compared to the results obtained from other methods.

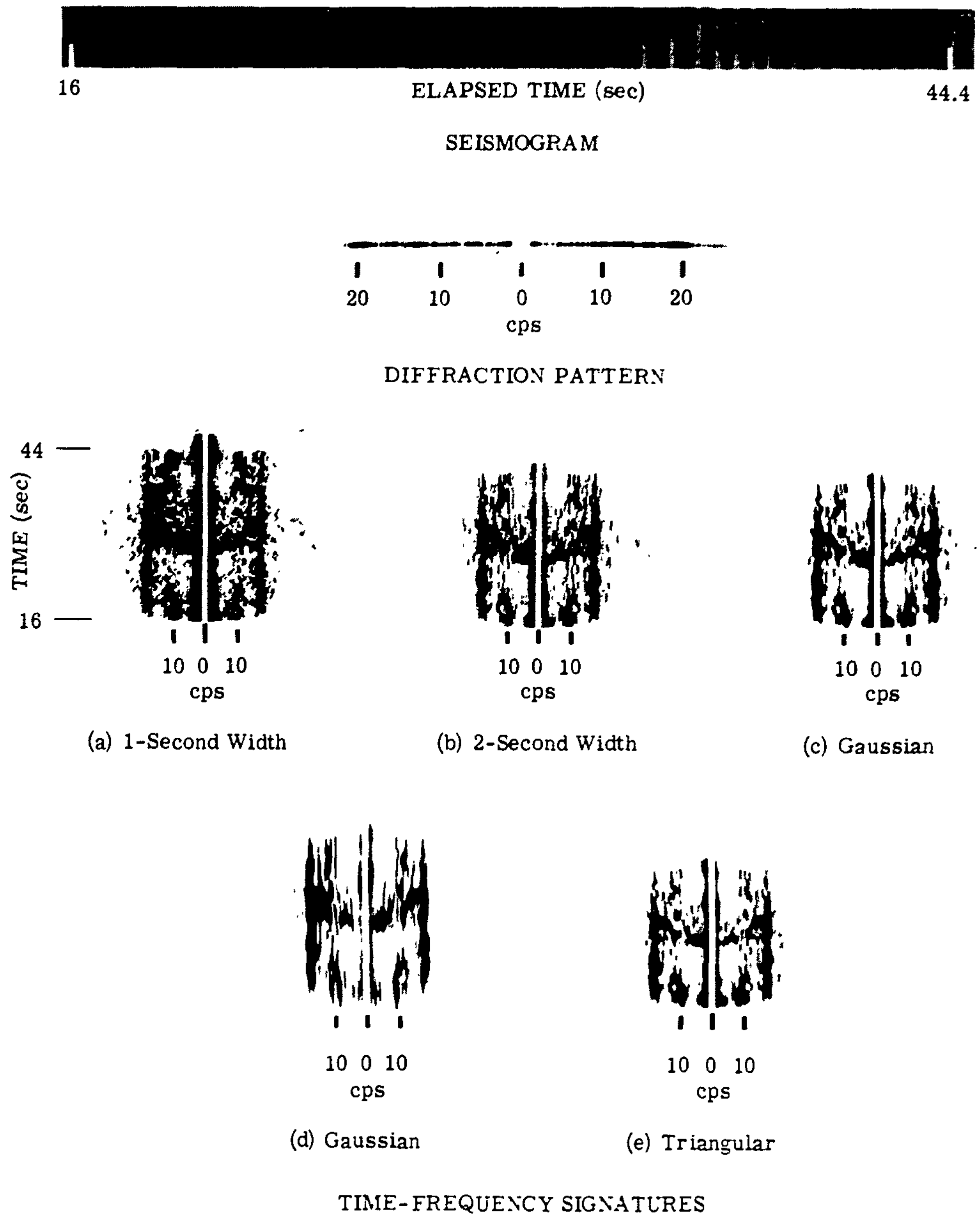


FIGURE 6. VARIABLE-DENSITY SEISMOGRAM, DIFFRACTION PATTERN OF ENTIRE SEISMOGRAM, AND TIME-FREQUENCY SIGNATURES OBTAINED WITH DIFFERENT SCANNING APERTURES. Each time-frequency signature was obtained with the aperture labeled with the same letter in Figure 4.

The recording was made 100 km from an explosion which occurred during construction of the St. Lawrence Seaway, between Messina, N. Y., and Cornwall, Ontario, on July 1, 1958. It was recorded on a vertical Willmore seismometer at Star Lake, N. Y.

In Figure 6 one can see the inception, change in frequency, and decay of various seismic waves. The fan-like structures at the bottom and top of the signatures are due to the opaque ends of the seismogram coming into view of the aperture. At the top, particularly noticeable with the rectangular apertures, a constantly narrowing slit is diffracted until the entire width of the aperture is crossed. At the bottom a widening slit is formed. Since distance from the central image is proportional to frequency, and the narrower the slit the higher-frequency components it contains, fanning in at the bottoms and out at the tops is explained. This effect is not seen so graphically when a Gaussian aperture is used, because the narrowest portions are lightly weighted.

To illustrate the method of quantifying time-frequency signatures, two microphotometer scans were made. Figure 7(a) shows a scan through time of the energy content at 10 cps, and Figure 7(b) shows a scan through frequency at 13 1/2 seconds after onset of the first P wave.

Note that these patterns are not exactly symmetrical, as would be expected from the theory. These figures are only illustrative of a method of quantifying time-frequency signatures, and not intended for actual measurement. Lack of symmetry can arise from focusing the camera slightly in front of or behind the Fourier plane, by slight tilting of the focal plane, by unequal distribution of light, by large wedging effects of the film base, and by lens aberrations. Most of these distortions can be and are being remedied.

One novel feature of this seismogram is the continuous energy at 18 to 20 cps. Little other comment need by made, since the conventional seismogram in Figure 4(a) can be compared visually to the time-frequency signatures.

6
DISPERSION SCANNING

Dispersion, caused by changes in the velocity of wave propagation with frequency, in turn causes frequency in a seismogram to change with time. Over a narrow frequency band frequency will change relatively slowly throughout a recorded wave train.

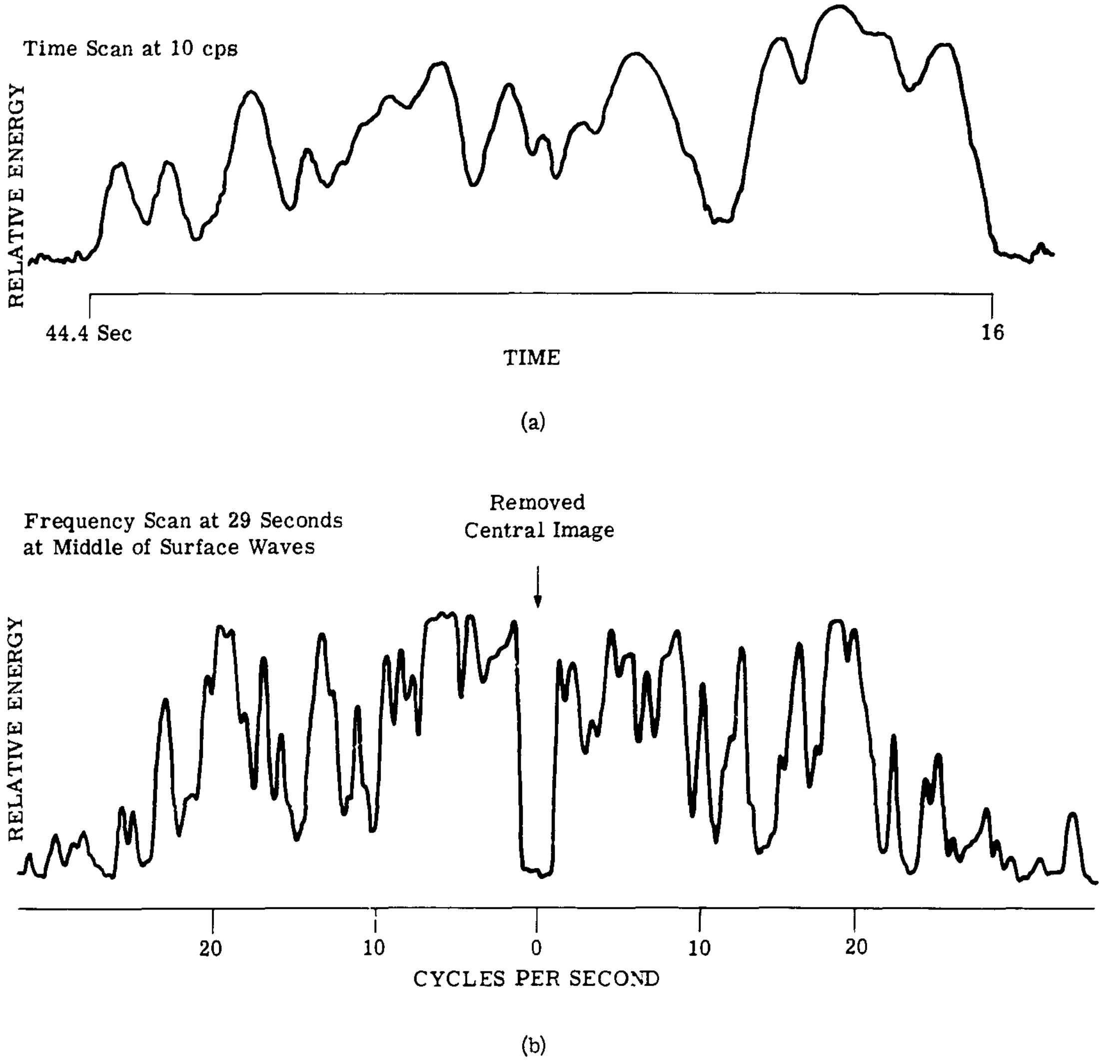


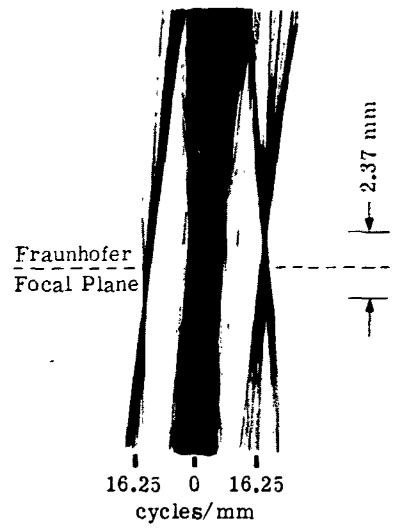
FIGURE 7. MICROPHOTOMETER SCANS. (a) Microphotometer scan across time-frequency signature (Figure 6) through time at 10 cps. (b) Similar scan through frequency at 13 1 2 seconds after onset of P wave. Diameter of circular scan aperture about 1/2 seconds in time or 1 cps in frequency.

In a variable-density seismogram, dispersion is shown by a slow change in spatial frequency. This change in the spatial frequency of an object causing diffraction will tend to focus light energy. This can be illustrated in two dimensions with the familiar zone plate. A narrow strip across and through the center of a zone plate approximates the form of a variable-density seismogram. The focusing effect of such a change in frequency causes diffracted energy concentrations in front of the Fraunhofer focal plane on one side of the central image, and behind it on the other side. The focal length changes occur because variable-density frequency changes act in the manner of lenses in combination with the Fraunhofer objective lens. One might view the resulting focal plane as tilted from the Fraunhofer focal plane. A treatment of this phenomenon can be found in an earlier report of this contract [5].

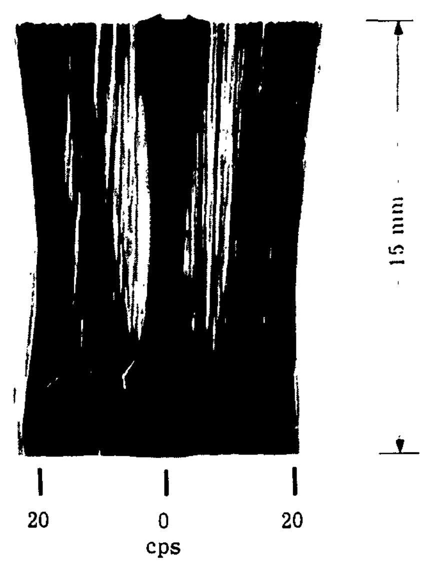
Scanning through the Fraunhofer focal plane, and recording changes in light energy concentrations, should reveal and define dispersion. An experimental setup similar to that used for obtaining time-frequency signatures can be employed. The aperture is widened so that the entire seismogram is diffracted. Then the light source is slowly brought through collimation by moving the collimating lens. Thus the focal plane of the diffraction pattern is slowly moved; this change is recorded by operating the viewing camera just as it is operated for the time-frequency signature. The resulting figure will represent frequency horizontally, and distance from focal plane vertically.

Consider a sinusoid recorded on variable-density film. Let it vary slowly and linearly in frequency; such a record can be made by recording the output of a laboratory audio oscillator while slowly changing its fine frequency adjustment. The diffraction pattern at the Fraunhofer focal plane will consist of a focused central image and two sidebands. each of a width corresponding to the lowest and highest frequencies in the record. At a position in front of the focal plane one sideband will be most concentrated; at the same distance behind the focal plane the other sideband will be most concentrated. In diffraction scanning through the focal plane three hourglass-like figures should form. The necks of the three hourglasses should locate the greatest energy concentrations of the two sidebands and the central image; the neck of the latter should be located at the midpoint between the necks of the sidebands. Figure 8(a) is a dispersion scan of the sinusoidal record treated in Figure 3. If no dispersion were present, each hourglass would have its neck in the Fraunhofer focal plane. where the neck of the central image is always located. Distance from the focal plane is related to the amount of dispersion. If the optical setup of Figure 1 were placed in a smoke chamber and viewed from above, one would expect the light reflections from the smoke to appear in the pattern of Figure 8.

Note the straight sides of the hourglass figures in Figure 8(a). Frequency change was linear. It is expected that the shape of the sides can indicate the nature of the dispersion by revealing optical caustics.



(a)



(b)

FIGURE 8. DISPERSION SCANS. (a) A sinusoid slowly changing in frequency. (b) Scan of the seismogram shown in Figure 5.

Focal displacements due to frequency changes in this record correspond to $\Delta\lambda/\lambda$ of 5.8×10^{-3}, where $\Delta\lambda$ is the change in wavelength per cycle, and λ is the average wavelength. The same focal displacements occur over a few wavelengths as over many. Therefore a very short portion of this record causes the same focal displacement that the entire record causes.

A dispersion scan of the previous seismogram is shown in Figure 8(b). Comparison will show that the slope of a diffracted "line" in Figure 4 corresponds with the distance of energy concentrations from the Fraunhofer diffraction plane in Figure 8.

The positions and lengths of wave trains can be indicated by dispersion scanning. If energy near a single frequency were found continuously throughout the seismogram, the light would converge from a large angle; that is, it would form a comparatively squat hourglass. If such energy were confined to a small section, light would converge from a correspondingly small angle. The direction of the angle bisector indicates the wave train's position in the seismogram, and hence its phase. This effect is aptly illustrated by the energy concentrations toward the edges of the diffracted hourglass in Figure 8(a). Little energy was transmitted from the middle portion of the record because of the defective mask.

7
COMMENTS ON TIME-FREQUENCY SIGNATURES

This method of optical scanning has been presented as a potential aid in diagnosing seismograms. The presentation has been illustrative; no measurements with known error-spread have been made. Only the conceptual basis and exploratory results of this method were intended for this memorandum. Quantified control and measurement of these processes is being pursued on this contract.

The "sound spectrograph" [6] is an identical output form to the time-frequency signature, and several other forms are similar. However, this optical metnod of obtaining such signatures has the advantage that the measuring parameters can be easily controlled and varied, and resolution is limited by the nature of the analyzing problem rather than by the measuring equipment. For instance, a Gaussian weighting, easy to construct by this method, would be difficult, if possible, to construct by means of an electrical filtering network.

Time-frequency signatures present only one channel of data—a major limitation in view of an optical system's capability of performing simultaneous analysis on many channels of data. An alternative method would be to scan many channels simultaneously while taking successive

photographs with a moving picture camera. Successive framing would lose the continuity for one channel, but would show channel comparisons. If the channels were array seismometer outputs, such comparisons would be valuable.

8
DISCUSSION OF FREQUENCY ANALYSIS BY MACHINE COMPUTATION

Because time-frequency signatures so vividly reveal the effects of the size and weighting of a scanning aperture, and emphasize the essentials of frequency analysis, discussion of (or speculation about) these effects appears appropriate in this memorandum. In particular, how can the judgment of an experienced seismologist be compared with the output of a computer relying on present-day mathematical formulations in frequency analysis? Though great effort is being expended on digital and analog computation, the relative merits of direct seismologist judgment and machine computation are seldom discussed; however, the question does cause controversy when it is raised. Of course it is understood that a computer may assist an experienced seismologist in his diagnoses.

It has been pointed out that, although much computer work is done, we have no computational method of identifying the onset and time duration of wave trains from a seismic disturbance. No one has developed a criterion or a method of attaching numbers to these wave trains. Yet individual seismologists are usually in agreement on the occurrence and nature of wave trains within individual seismograms.

As shown in Equation 2, frequency and time resolution oppose each other in Fourier analysis. This opposition is directly related to, if not identical with, probably the most sacrosanct principle in modern physics — Heisenberg's uncertainty relation. That is, the equation

$$\Delta f \Delta t = k \tag{1}$$

has the same form and is derived in precisely the same manner as Heisenberg's uncertainty relation

$$\Delta p \Delta q \gtrsim h \tag{3}$$

Indeed, Schrödinger shows conceptually [7, p. 157] how Equation 3 follows directly from Equation 1.

We know, then, that the shorter the interval of time the less finely we can analyze a time frequency. A reason may lie in the dominant position of the boundary conditions, which in our

particular case are set by the scanning aperture. Mathematically they are the integral limits. The finite Fourier transform does not discriminate between the function and its boundaries. In fact, it will indicate a pure harmonic only if $\Delta t \rightarrow \infty$, that is if the time function is unbounded. But a function seen through an aperture has artificial boundaries, which, unfortunately, are included in the analysis.

As an example, consider Figure 9. We have drawn here the curve of a pure harmonic, and arbitrarily truncated it so that it is only a few wavelengths long. The Fourier transform would indicate that the figure has a wide frequency band. If this curve were viewed through a small aperture, as shown in the lower part of Figure 9, the Fourier frequency analysis would be more blurred, and contain the frequency with higher orders corresponding to the wavelength of the aperture. Furthermore, the analysis is dependent on both the width and the shape of the aperture. Therefore, an infinite number of time frequency signatures, corresponding to an infinite number of shapes and sizes of apertures, can be obtained from the same function. The usual presentation of results of running frequency analysis without specifying the aperture is at best a simple omission, and at worst misleading.

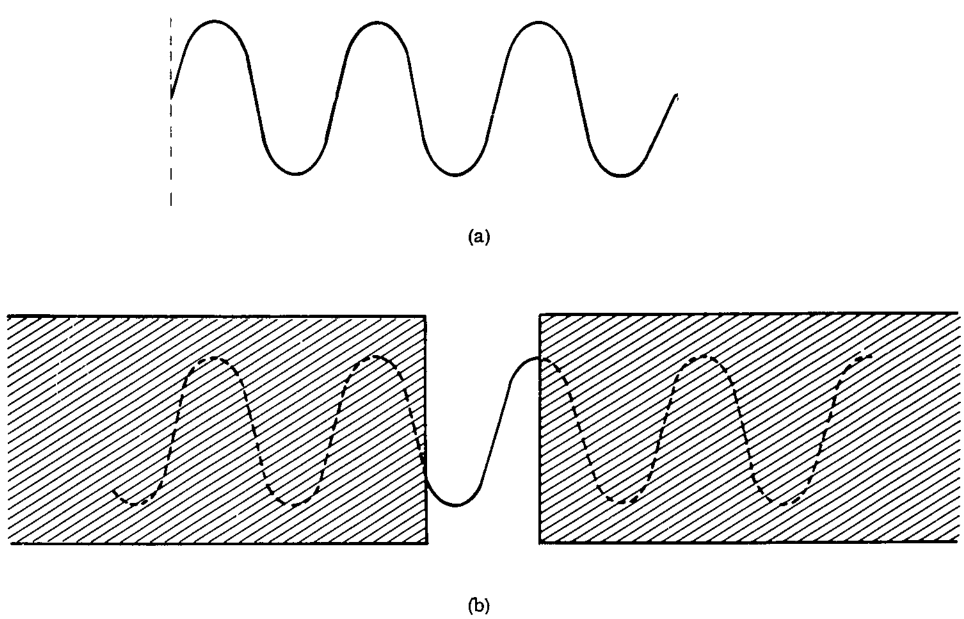


FIGURE 9. REPRESENTATIONS OF PURE HARMONIC. (a) Truncated representation. (b) Representation as viewed through an aperture.

The cruxes of this discussion are that (1) in conventional analysis the artificial boundaries of the function are treated exactly as is the function itself, and (2) by varying aperture scanning an infinite number of analyses can be obtained.

Now consider the human being. Again look at Figure 8. The description of a pure harmonic of a few wavelengths offends no one who hasn't been indoctrinated through Fourier analysis. We can say, "Within these boundaries the function has the shape of a pure harmonic." In fact, this can be said about the partial wavelength viewed through the small aperture. Under the spell of Fourier analysis one would say, "Including these limits the Fourier transform of the function has the size of a wide band, and through the small aperture it is a mess." One difference, an overpowering one in seismology, is that a man can identify boundaries, relegate them to their proper place, and concentrate on the shape of the curve within these boundaries. The onset, duration, and decay of wave trains are easier to identify when they can be separately identified and related.

This difference enables a man to see the abruptness of a change (or boundary) and accept the abruptness apart from the function, so that in effect he achieves finer frequency resolution than that available through the Fourier transform.

An analogy with a standing wave in a string can be made. Let the ends of a string be attached to widely separated rigid objects. Consider the curve described by standing waves in the string at any instant. We can analyze this curve by taking the junctions of the string ends at the rigid objects as boundary conditions. Now visually block out the string with two opaque curtains, except for some distance around its midpoint. A man will see through the aperture between the curtains, and realize that the boundaries of the string are to the right and left of the edges of the curtain. In our present use of computer analysis for this case (including the computer described here) the two edges of the curtain would be taken as boundaries of the function described by the string.

The above is speculation, of course. If it is true a question can be raised about the adequacy of the Fourier transform as currently employed. Also, if $\Delta f \Delta t$ is directly analogous to $\Delta p \Delta q$ in the uncertainty principle about which physicists feel most certain, then possibly nothing can be done to obtain frequency resolution by machine or analysis to compare with that achieved by humans (again, if our thesis is true).

Although it is generally treated as exact, the analogy of $\Delta f \Delta t$ in Fourier analysis to $\Delta p \Delta q$ in the uncertainty principle may not be exact, because the latter refers to simultaneous knowledge of p and q for use as an initial condition in future calculations, as explained by Heisenberg

[8, p. 20]. Fourier analysis of seismograms may be made for a purpose quite different in principle from that of determining an initial condition of a particle, since we no longer care about the time interval after the frequency has been determined. As a matter of fact, Δt is not even mentioned in most presentations of running frequency analysis.

To continue the speculation. If humans achieve better resolution, what kind of analysis could duplicate it? The boundary discrimination achieved by humans does not appear to have been investigated analytically. As noted above, an infinite number of analyses can be made by varying the size and shape of the scanning aperture. Yet to this writer's knowledge, gained from literature search and conversation with contributors in this field, the approach toward obtaining finer frequency resolution by comparing the results from two or more apertures does not seem to have been investigated. Possibly humans perform aperture variations and comparisons intuitively. Certainly they know that the edge of the aperture is not part of the function, and seek to dissociate it from the function. Varying the aperture corresponds mathematically to varying the limits and the weighting in the Fourier integral. Again, such variation is not done in either the Fourier transform or the rigorous derivation of Heisenberg's uncertainty principle.

Nothing conclusive is intended in this discussion. There is some concern about the relative merits of man's judgment and machine computation in seismogram frequency analysis. The writer tends to believe that human judgment might have some advantages over present-day frequency analysis, and has suggested a possible reason for this. Also, if the human is better the analysis method he uses should be developed mathematically. This matter is directly related to the analytical method presented in this memorandum, in that diffractive computation provides a convenient means for confirming experiments.

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